中國機械工程學會第二十一屆全國學術研討會論文集
中華民國九十三年十一月二十六日、二十七日 國立中山大學 高雄・台灣

A Study upon Sub-Pixel Accuracy of Point Identification between Digital Images

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ABSTRACT

An idealized model on re-constructing digital image between coarse and fine pixel image is proposed. The maximum local cross-correction coefficient method is employed to identify the associated point of an digital image with respect to another digital image. The distance between two corresponding points is considered as the displacement between points. After comparing this distance to the exact distance, the identification error can be defined. It is found that the following methods can improve the error on identification: scale up the gray level value by an amount approximately equal to the maximum gray level of two images; construct the fine pixel image via the monotonic cubic spline interpolation to perform the fine grid identification; and employs a modified Shepard interpolation to perform selected interpolation which excludes all points with an extra-ordinary displacement. The linear conservative interpolation method is examined and is found to have a small effect of improving identification error because the proposed method of calculating the gray level gradient is improper. Ocassionally, it is helpful to improve image visibility. A motor plate image was employed to test the applicability of the proposed model. The result shows that only the gray level scale up and the selected Shepard interpolation are effective.

Keywords: digital image reconstruction, sub-pixel identification error improvement, interpolations.

1. Introduction

Because of the rapid development of computer hardware and software, the image processing has become a well developed technique such that some commercial software packages are available. The overall information can be easily grasped from text book [1]. Among many applications of the image processing, people had employed the digital image to reconstruct three-dimensional surface sophisticatedly any have many practical applications. Since it can be programmed via a software package, it has the potential of collecting and treating many huge data sets simultaneously. Unfortunately, it has many unknown properties introduced by the limited digits of the gray level and limited pixels. Therefore, it is worth to study these fundamental issues.

When one reconstructs a there-dimensional surface, he grasps data from several digital images. As comparing two pictures, he needs to identify which point of the second picture is corresponding to a pixel of the first picture. It seems that the accuracy of this identification procedure is the key technology of the application of reconstructing surface from digital images. Since both the spatial resolution and gray level are limited, to improve the accuracy should be done in the sub-pixel level. The main concerns of this study are to examine several methods of improving sub-pixel identification.

In order to simplify the problem, a idealized model is employed to construct a reference picture from an existing digital image. In other words, every point of the existing picture has a known location on the reference picture. Consequently, the identification error of any method can be easily calculated. Basing on the ideal model of constructing a picture, the following techniques are examined: sub-pixel interpolation, conservative interpolation, and the modified Shepard interpolation.

2. Theoretical Analysis

2.1 Ideal Model of Collecting Information in a Pixel

Assume that all the scattering, reflection and other factors passing through the sampling aperture and insufficient sensitivity of the screen converting the light into gray level are neglected so that the light exhibits a geometric optics behavior. Moreover, the range of screen of a pixel is considered as a rectangle. Consequently, when the image with a coarse pixel is converted into an image of fine pixel, the pure geometrical mapping can be employed. For example, let rectangles A and B be two pixel ranges of the coarse pixel image and rectangle C to be a pixel range of the fine pixel image, and $C \subset (A \cup B)$, their areas and gray levels are A_A, A_B, A_c, f_A, f_B, and f_C, respectively. Noted that these gray levels are uniformly distributed over the whole pixel because of the character of a digital image. The gray level of pixel C is equal to $(A_A f_A + A_B f_B)/A_c$. All the other rule of gray level calculation are in a similar manner.

2.2 Monotonic Cubic Interpolation

For the sub-pixel interpolation, many literatures employ the polynomial interpolation [1] because the induced oscillation does-not introduce error in most digital image processing. For a sub-pixel identification problem, on the other hand, the Runge oscillation may introduce a large error so that the polynomial interpolation is not a suitable method. In this study, the cubic monotonic interpolation of Hyunk [2] is employed which imposes the Essential Non-Oscillatory limiter to the first order derivative, say

$$y(x) = c_3(x-x_i)^3 + c_2(x-x_i)^2 + c_1(x-x_i) + c_0$$
$$c_0 = y_i, c_1 = y_i^{'}$$
$$c_2 = \frac{3s_{i+1/2} - 2y_i^{'} - y_{i+1}^{'}}{x_{i+1} - x_i}, c_3 = \frac{y_i^{'} + y_{i+1}^{'} - 2s_{i+1/2}}{(x_{i+1} - x_i)^2}$$
$$s_{i+1/2} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

$$y_i^{'} = \text{sgn}(t_i)\min[\frac{1}{2}\left|p_{i-1/2}^{'}(x_i) + p_{i+1/2}^{'}(x_i)\right|, \max(3\left|s_i\right|, \frac{3}{2}\left|t_i\right|)]$$
$$p_{i-1/2}^{'}(x_i) = s_{i-1/2} + d_{i-1/2}(x_i - x_{i-1})$$
$$p_{i+1/2}^{'}(x_i) = s_{i+1/2} + d_{i+1/2}(x_i - x_{i+1})$$
$$t_i = \min\text{mod}[p_{i-1/2}^{'}(x_i), p_{i+1/2}^{'}(x_i)]$$
$$= \text{sgn}[p_{i-1/2}^{'}(x_i)]\min[\left|p_{i-1/2}^{'}(x_i)\right|, \left|p_{i+1/2}^{'}(x_i)\right|], \quad if \quad p_{i-1/2}^{'}(x_i) \times p_{i+1/2}^{'}(x_i) \geq 0$$
$$= 0, otherwise$$
$$d_{i+1/2} = \min\text{mod}(d_i, d_{i+1}), \quad d_i = \frac{s_{i+1/2} - s_{i-1/2}}{x_{i+1} - x_{i-1}}$$
$$s_i = \min\text{mod}[s_{i-1/2}, s_{i+1/2}]$$
$$\text{sgn}(a) = 1, \quad if \quad a > 0$$
$$= 0, \quad if \quad a = 0$$
$$= -1, \quad if \quad a < 0$$

This interpolation is employed to generate fine gray level distribution along horizontal line running across the cell center of pixel. Then, along all the fine points which forms vertical lines, the same method is applied to generate fine gray level distribution of the whole digital picture.

2.3 Conservative Interpolation

Consider the schematic diagram shown in Fig.1, the cutting edge is designed as line 7-8 and divides the pixel cell 12341 into two zones one as the high gray level zone and the other the low gray level zone. The slope of the cutting line is determined by the following relations [3-6]

$$\nabla f_{m,n} = \frac{\partial f(x_{m,n}, y_{m,n})}{\partial x}\vec{i} + \frac{\partial f(x_{m,n}, y_{m,n})}{\partial y}\vec{j} \tag{3}$$
$$dy/dx \cdot \nabla f_{m,n} = -1$$

For a conservative interpolation, the resulting gray level distribution should satisfies the following relation.

$$f_{m,n}\Delta x \Delta y = \iint_{A_L} f_L(x,y)dA + \iint_{A_H} f_H(x,y)dA \tag{4}$$

If the situation of Fig.1 is chosen, this equation becomes

$$f_{m,n}\Delta x\Delta y = \frac{A_{125}}{3}(f_{1H} + f_{2H} + f_{5H}) + \frac{2A_{157}}{3}(f_{1H} + f_{5H} + f_{7H} + f_{1H} + f_{7H} + f_{8H}) + \frac{A_{346}}{3}(f_{3L} + f_{4L} + f_{6L}) + \frac{2A_{367}}{3}(f_{3L} + f_{6L} + f_{7L} + f_{6L} + f_{7L} + f_{8L}) \tag{5}$$

where f_{jL} and f_{kH} are evaluated from

$$\begin{aligned} f_L &= f_{\min} + (f_x)_{\min}(x - x_{\min}) + (f_y)_{\min}(y - y_{\min}) \\ f_H &= f_{\max} + (f_x)_{\max}(x - x_{\max}) + (f_y)_{\max}(y - y_{\max}) \end{aligned} \tag{6}$$

$f_{\max}, (f_x)_{\max}, (f_y)_{\max}$ and $f_{\min}, (f_x)_{\min}, (f_y)_{\min}$ are data at points where $|\nabla f|$ achieves its maximum and minimum values around the pixel cell. Noted that the location of the cutting line is unknown so that an unknown parameter t is employed to specify points 7 and 8 via the following equations

$$\begin{aligned} x_8 &= x_1, \quad y_8 - y_1 = t(y_6 - y_1) \\ x_7 &= x_2, \quad y_7 - y_5 = t(y_3 - y_5) \end{aligned} \tag{7}$$

After substituting several proper relations into Eq.(5), it becomes a cubic polynomials of t, the solutions and Eq.(7) will give the coordinates of points 7 and 8. Finally, the fine pixel gray level distribution within the cell is evaluated by Eq.(6).

Since the cutting edge may cut the pixel at any location in the three different location, say the triangle 125, parallelogram 1536, or triangle 346, this study includes all these possibilities. Moreover, the slope of the cutting may have 8 different situations so that the total possible arrangements are of 24 types. In this study, the above mentioned monotonic interpolation is normally employed and switches to this conservative interpolation whenever $|\nabla f_{m,n}| > \alpha$, where α is an user specified parameter.

2.4 Seleted Shepard Interpolation

Since the predicted point movement from one picture to another may have isolated large error point, the Shepard interpolation is modified to be [7]

$$\Delta\vec{r}_{p,q} = \left[\sum_{\substack{i,j\in D \\ |\Delta\vec{r}_{i,j}|\le k}} \Delta\vec{r}_{i,j}/d_{i,j}\right] \div \left[\sum_{\substack{i,j\in D \\ |\Delta\vec{r}_{i,j}|\le k}} 1/d_{i,j}\right], \ \textbf{if} \ \ \Delta\vec{r}_{p,q} > k$$

$$d_{i,j} = \sqrt{\left(\vec{r}_{p,q} - \vec{r}_{i,j}\right)^2}, \ \ k = \alpha \sum_{i,j\in D_1} \Delta\vec{r}_{i,j}/L \tag{8}$$

This selected interpolation replaced the point displacement $\Delta x, \Delta y$ at point (p,q) by the surrounding points' data but excluding those points with a larger displacement $\sqrt{\Delta x^2 + \Delta y^2} > k$.

2.5 Local Cross Correlation Coefficient

For the point (i,j)-th point of one picture, the location where the following cross correlation coefficient [8-12] achieves a local maximum value is considered as the mapping point of (i,j) on the second picture.

$$\rho_{12}(i,j;k,l)=\frac{1}{S}\sum_{p,q\in D_2}\exp[-\{(x_{i+p,j+q}-x_{i,j})^2+(y_{i+p,j+q}-y_{i,j})^2\}/2\sigma^2]\cdot f^1_{i+p,j+q}f^2_{k+p,l+q}$$

$$S=\sum_{p,q\in D_2}\exp[-\{(x_{i+p,j+q}-x_{i,j})^2+(y_{i+p,j+q}-y_{i,j})^2\}/2\sigma^2]\cdot \mathrm{Max}[f^1_{i+p,j+q},f^2_{k+p,l+q}]^2 \quad (9)$$

in which the indices k,l denotes the point indices of the second picture, range D_2 takes a rectangular shape, and the Gaussian kernel is employed to fading the undesired boundary effect of a uniform weighing formula.

2.6 Procedure of Estimating Identification Error

For the sake of completeness, the procedure is summarized below:

1. Construct a new picture from an existing picture.
2. Construct fine pixel pictures for two pictures via the monotonic interpolation.
3. Calculate the local cross correction for a point (i,j) of the first picture and points of the second picture. Choose the point with a maximum correlation coefficient as the reference displacement of a point (i,j) to a new location on the second picture.
4. Comparing the estimated displacement and the prescribed displacement to obtain the identification error.
5. Check the effect of all improvements.

3. Results and Discussion

The first test case assumes a gray level distribution of one picture as (shown as Fig.2)

$$f(x,y)=1.1+\sin(2\pi x)\sin(2\pi y)+u(x-0.6)-u(x-0.4)+u(y-0.6)-u(y-0.4) \quad (10)$$

where $u(.)$ is the unit step function and the constant 1.1 is employed to improve accuracy. Then the image moves according to the rule

$$x=t-0.2t^2,\quad y=1.6t+1.3t^2 \quad (11)$$

and forms the second picture as shown in Fig.3.

As to find the local maximum of the cross-correlation coefficient, consider the (21,25)th point of Fig.2, the corresponding correlation coefficients of different point of Fig.3 are shown in Table 1. Every coefficients are evaluated by a 10 by 10 rectangular range with $\sigma=0.4\Delta x$, $\Delta x=0.025$. From the table, it is obvious that the location of this point on Fig.2 is at $(x,y)=(0.5375,0.6625)$ by the coarse pixel calculation. After comparing with the location predicted by Eq.(11), say (0.54752, 0.65363), the identification error is (0.02002,-0.00907).

Fig.4 shows the local correlation coefficient without the constant 1.1 in Eq.(10), while Fig.5 is that with 1.1. It is seen that variation of Fig.4 is much rapid than that of Fig.5. Since a rapid variation of coefficient frequently gives a larger identification error than that with a smooth distribution, the scale up rule is a necessary technique to improve identification accuracy. The resulting identification error is shown in Fig.6. When the fine pixel calculation is made via the monotonic cubic interpolation, the error distribution becomes that shown in Fig.7 which clearly reduces the identification error. Because the boundary correction is not done in this study, the error around the right boundary is very large and will be treated in a further study. When the selected Shepard interpolation is employed with $k=0.075$ in Eq.(8) and range D is a 4 by 4 rectangle, the result is shown in Fig.8 which further improves the error. When the conservative interpolation is employed, the resulting digital pictures are shown in Fig.9. The original image's edge becomes very sharp, while the moved image has some imperfections which needs further improvement upon the cutting edge prediction. The resulting identification is shown in Fig.10 which improves the error slightly in spite of the deteriorated moved picture. It seems that, if the cutting edge prediction becomes more effective, the conservative interpolation can play an important role too.

The second test is one part of a real digital image of the motor cycle plate as shown in Fig.l1. After employing Eq.(11) and the idealized reconstruction procedure, the resulting digital image is shown in Fig.12. Note that, both Figs.11 and 12 use 256 finite gray level to display the image. The resulting coarse and fine pixels' identification errors are shown in Figs.13 and 14, respectively. It is seen that the finite digit resolution of the gray level does deteriorate the error performance. However, the selected Shepard interpolation can further improve the error as shown in Fig.15. The resulting images of using the conservative interpolation are shown in Fig.16. Now the visibilities of these images are partially improved in somewhere but become worse in else where because the present conservative interpolation reforms the gray level with a single pixel only. The final identification error is shown in Fig.17 which is only slightly better than that of Fig.15.

The above discussions show that, if the gray level is of infinite digital as in the first test case, the gray level scale up by a factor approximately equal to maximum gray level, monotonic fine pixel interpolation and the selected Shepard interpolation can improve the identification error effectively. The conservative interpolation method has the similar potential too provided that the cutting edge prediction is properly done. On the other hand, when the gray level is of 256 levels, the gray level scale up method and selected Shepard interpolation can improve the error. The monotonic fine pixel interpolation and conservative interpolation may or may not have positive effect of reducing identification error. Therefore, it points out that, a further study of recovering the gray level resolution is necessary.

4. Conclusions

An idealized model of reconstruct a digital image is proposed to check the effect of identification error by some improvement method. When the resolution of the gray level has nearly infinite many digits, most proposed improvement methods are effectively. If there is only 256 gray level, only two improvements are effectively. Therefore, a further study is necessary.

Acknowledgement

This work is supported by the National Science Council of Taiwan under the grant number NSC-92-2212- E006-077.

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Table 1 Local cross-correlation coefficients

	i=17	18	19	20	21	22	23	24	25
j=21	.547	.636	.726	.807	.880	.842	.747	.661	.599
22	.552	.644	.737	.821	.900	.860	.759	.669	.604
23	.562	.655	.750	.836	.920	.878	.774	.683	.616
24	.587	.679	.774	.863	**.954**	.913	.806	.713	.646
25	.615	.696	.777	.853	.934	.907	.818	.738	.678
26	.640	.698	.752	.804	.864	.853	.797	.745	.705
27	.666	.696	.725	.755	.796	.797	.772	.750	.731
28	.681	.689	.698	.711	.739	.748	.745	.745	.746
29	.679	.672	.668	.672	.691	.703	.711	.724	.737

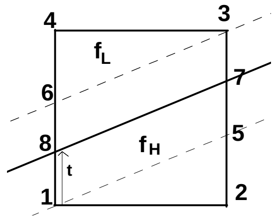

Fig.1 The schematic diagram of the sharp edge (line 7-8) of the rectangular cell 12341.

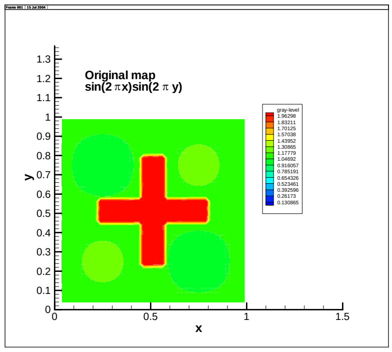

Fig.2 The first digital image 40 by 40 pixels.

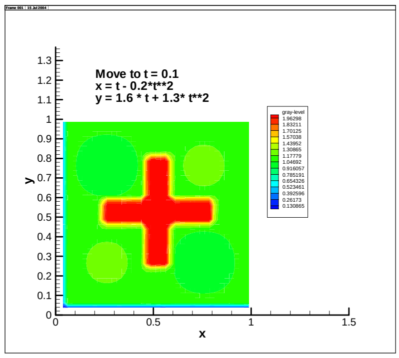

Fig.3 The second digital image constructed by the idealized model, 40 by 40 pixels.

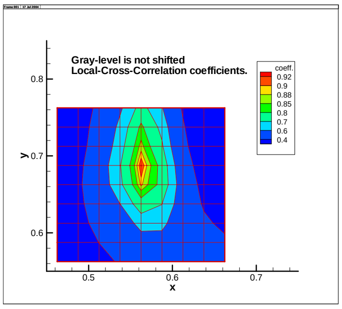

Fig.4 The cross-correlation coefficient distribution of the (21,25)th point without the factor of 1.1 in Eq.(10).

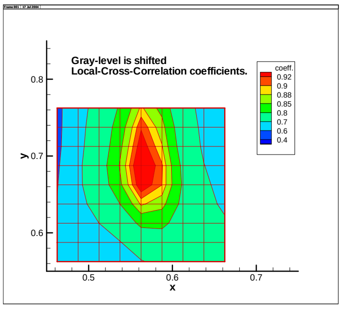

Fig.5 The cross-correlation coefficient distribution of the (21,25)th point with the factor of 1.1 in Eq.(10).

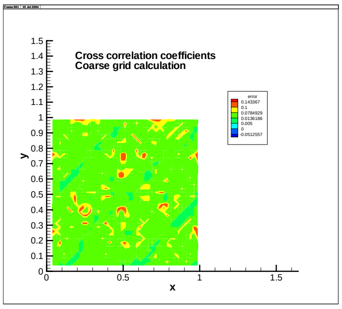

Fig.6 The coarse pixel identification error.

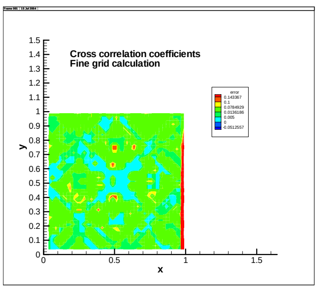

Fig.7 The fine pixel identification error.

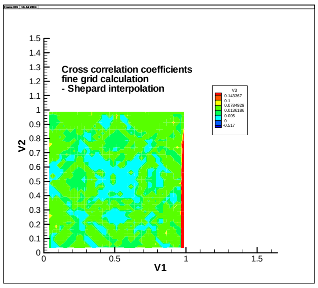

Fig.8 The identification error improved by using the selected Shepard interpolation to Fig.7.

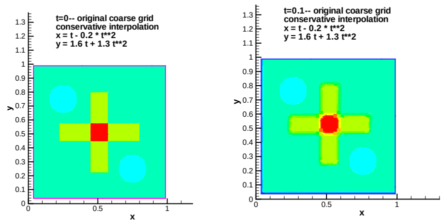

Fig.9 The resulting digital image by conservative interpolation: the left is original picture and the right is the moved picture.

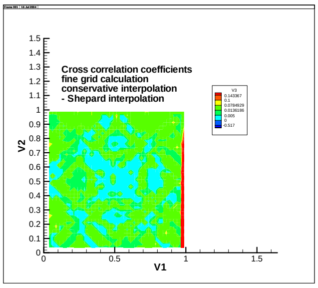

Fig.10 The fine pixel identification error improved by the conservative interpolation and the selected Shepard interpolation to Fig.7.

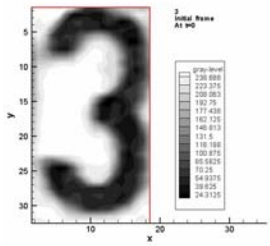

Fig.11 One part of the digital image of a motor cycle plate.

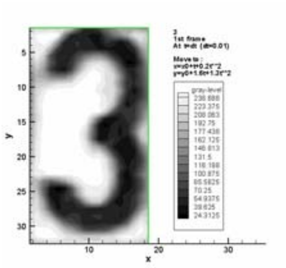

Fig.12 One part of the digital image of a motor cycle plate after moving according to Eq.(11) and reconstructed by the idealized model.

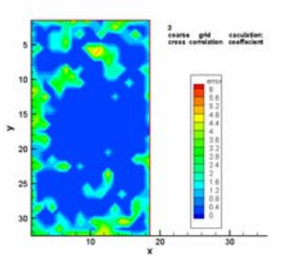

Fig.13 The coarse pixel identification error.

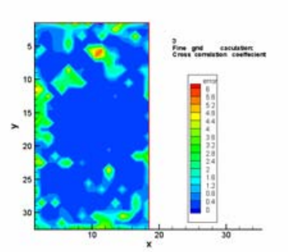

Fig.14 The fine pixel identification error.

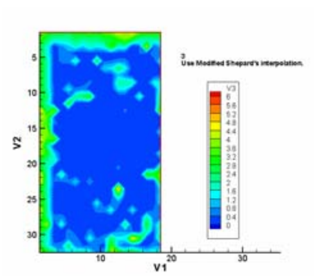

Fig.15 The identification error improved by using the selected Shepard interpolation to Fig.13.

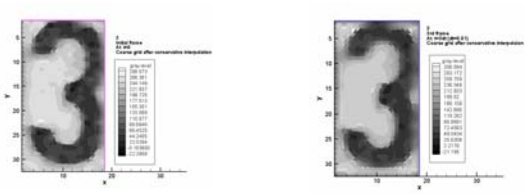

Fig.16 The reconstructed moved image via the conservative interpolation: the left is original picture and the right is the moved picture.

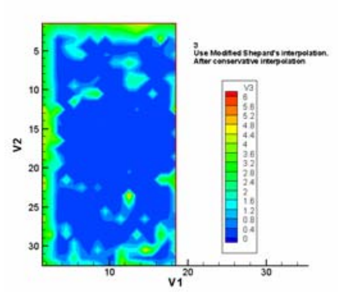

Fig.17 The fine pixel identification error improved by the monotonic fine pixel interpolation, conservative interpolation and the selected Shepard interpolation to Fig.16.

數位影像間之次像素定位精確度的初步探討

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摘要

本文發展一種數位影像在粗細像素之間重建數位影像的理想化模式。將一設定物件做短距離位移後，構建成兩張理想化的數位影像，再從第一張圖之點，計算在第二張圖上各點之局部互相關係數，最後取系數極大時之位置，當作在第二張圖上對應於第一張圖該點之位置。定位誤差是由理論位置和估算位置之差異的距離決定。本文發現下列三種方法可以明顯的降低定位誤差：將灰階值加上最大灰階值之均勻性平移，使用單調性分段三次曲線內插法建立細像素後，進行次像素估算定位，和使用修正型之選擇性 Shepard 內插法。本文亦討論線性守恆性內插法，發現前者可以略為改進定位誤差有時也有助於影像的可視性，而後者則在細像素估算有潛力。本文亦應用到數個車牌影像的實例上，以討論本文的模式實用之可能性。

關鍵詞:理想化數位影像重建,次像素定位誤差,內插法,局部相關係數