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題目：An True Short Time Fourier Transforms For A Time Series Data String

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若投稿論文為國科會研究計畫之成果，請註明國科會計畫編號：
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時變數據的二維短時間間格傅數轉換法之研究

An True Short Time Fourier Transforms For A Time Series Data String

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ABSTRACT

The simple Fast Fourier Transform (FFT) algorithm with a small spectrum error is employed to construct the two-dimensional plot of the true short time Fourier transforms. Before the data string is treated by the FFT algorithm, the iterative filter via the Gaussian smoothing is applied to remove the undesired non-sinusoidal part and those wave components remote from the interested wave. The Fourier spectrum is evaluated in a window whose end points are defined by zero crossing positions of the original data. The resulting spectrum is assumed to be valid at the central point of the window. Consequently, a two-dimensional data set defined by these central points and spectrum is constructed. This two-dimensional plot is a more direct approach than the Gabor transform, which has already been named as the short time Fourier expansion. A test case examines the beat wave formed by 3 known sine waves and shows that the zero crossing point can not be put at region where the approximate amplitude of the beat attains local minimum value. Two test cases are employed to show the capability of capturing local properties of this new time-frequency analysis. It seems that, because of the uncertainty of the time-frequency analysis via the Fourier series expansion and the non-uniform window width, the resulting time-spectrum plot evaluated via a relative short window width arrangement shows a faded character in spite of the high capability of capturing the local property. For a sufficiently large window size, the plot shows a steady property but lose the ability of capturing local property.

Keywords: FFT with small error, Time Frequency Analysis.

1. INTRODUCTION

Because of the rapid development of computer hardware and software, the capability of collecting huge number of long data string increases rapidly. Generally, a real data string frequently has a complex structure which changes main characters rapidly. Consequently, the classical Fourier spectrum can not fulfill the desire of recognizing all the detailed information. In order to look into the local property, both the wavelet and Gabor transforms are widely applied as tools of the time-frequency analysis [1,2]. Although both wavelet and Gabor transforms have being extensively studied from mathematical point of view, these methods are approximate expansions rather than a exact mapping procedure such as the Fourier spectrum. Therefore, it is interesting to restudy the property of a true short time Fourier transformation.

For a real data string, the Fourier spectrum evaluated by the current Fourier expansion may involve significant low frequency error because of the non-periodic condition at two ends. The first step to remove the error, in a previous study [3], the cubic moving least squares method is employed to remove non-sinusoidal and low frequency parts. Second, two zero crossing points next to two ends are fixed by a interpolation procedure. Third, a modification upon the monotonic cubic spline interpolation method [4] was made and it was employed to redistribute data point with the specification that total number of points is equal to an 2^m and at least 2 new points are embedded into the original data window. Finally, before the FFT algorithm is employed, an odd function extension of the data string is made. The resulting data string has the periodicity of the data and all resolved derivatives.

In two following studies [5,6], the iterative filter (basing on the Gaussian smoothing and cubic moving least squares method) without phase error was proposed. If wave components of a data string have a obvious gap on spectrum domain, the filter can effectively give the low and high frequency part. In other words, the cubic moving least squares method employed in Ref.[3] may leave certain low frequency wave component on the sinusoidal part.

This study will employ the iterative filter of Ref.[5,6] to remove the non-sinusoidal and low frequency parts. Moreover, because of the FFT algorithm gives a spectrum with small spectrum error, it will be employed to obtain a true short time Fourier series expansion and develop a time-spectrum plot. It should be noted that, in general, the original combination of non-sinusoidal and low frequency parts are not known. As a consequently, the separated high frequency part may be incorrect to certain degree and will be discussed in this study too.

2. ANALYSIS

The Iterative Filter Basing on Gaussian Smoothing

Assume that a discrete data string can be approximated by

$$y(t) = \sum_{n=0}^{N} b_n \cos\left(\frac{2\pi t}{\lambda_n}\right) + c_n \sin\left(\frac{2\pi t}{\lambda_n}\right) \tag{1}$$

In Ref.[5,6], it was proven that after applying the Gaussian smoothing once, the resulting smoothed data becomes

$$\bar{y}_1(t) \approx \sum_{n=0}^{N} a(\sigma/\lambda_n)\{b_n \cos\left(\frac{2\pi t}{\lambda_n}\right) + c_n \sin\left(\frac{2\pi t}{\lambda_n}\right)\} \tag{2}$$

where $a(\sigma/\lambda_n)$ is the attenuation factor introduced by the smoothing and can be proven numerically that

$$0 \le a(\sigma/\lambda_n) \approx \exp[-2\pi^2\sigma^2/\lambda_n^2] \le 1 \tag{3}$$

If the removed high frequency part is denoted as $y_1^{'}$ and apply the same smoothing to it to obtain the second smoothed result as $\bar{y}_2$ and repeat the same procedure to obtain the $m-$th smoothed and high frequency part as $\bar{y}_m$ and $y_m^{'}$, respectively. The following relation can be built

$$y_m^{'} = \sum_{n=0}^{N} [1 - a(\sigma/\lambda_n)]^m \left[b_n \cos\left(\frac{2\pi t}{\lambda_n}\right) + c_n \sin\left(\frac{2\pi t}{\lambda_n}\right)\right]$$

$$\begin{aligned} \bar{y}(m) &= \bar{y}_1 + \bar{y}_2 + \dots + \bar{y}_m = y - y_m^{'} \\ &= \sum_{n=0}^{N} A_{n,m,\sigma}\left[b_n \cos\left(\frac{2\pi t}{\lambda_n}\right) + c_n \sin\left(\frac{2\pi t}{\lambda_n}\right)\right] \end{aligned} \tag{4}$$

$\bar{y}(m)$ can be considered as the smoothed part and $y_m^{'}$ as the high frequency part. After substituting Eq.(3), the attenuation factor of employing the Gaussian smoothing method satisfies

$$\begin{aligned} &A_{n,m,\sigma} = 1 - [1 - \exp\{-2\pi^2\sigma^2/\lambda_n^2)] \\ &0 \le A_{n,m,\sigma} \le 1 \end{aligned} \tag{5}$$

Obviously, the accumulated smooth part is embedded by a diffusive attenuation factor without phase error. It was also proven in Ref.[5,6] that the transition region from $A_{n,m,\sigma} = 0$ to 1 is much narrower than that of the original $a(\sigma/\lambda_n)$ for a sufficiently large iteration step m.

Suppose that all the waveforms within the range of $\lambda_{c1} < \lambda < \lambda_{c2}$ are insignificantly small. The above mentioned iterative smoothing procedure can be an effective filter to give both the low and high frequency parts. The desired parameters σ and number of iteration steps m are solved by the following simultaneous equations.

$$\begin{aligned} &1 - [1 - \exp(-2\pi^2\sigma^2/\lambda_{c1}^2)]^m = B_1 \\ &1 - [1 - \exp(-2\pi^2\sigma^2/\lambda_{c1}^2)]^m = B_2 \end{aligned} \tag{6}$$

where $B_1, B_2 = 0.01,\ 0.99$ are employed in this study.

Before the FFT algorithm of Ref.[3] is employed. One can use a classical FFT algorithm to obtain a spectrum. After inspecting both the original data and spectrum, he can decide a suitable transition range defined by λ_{c1} and λ_{c2} to remove the non-sinusoidal and low frequency part. In this study, the filter is also repeatedly employed to give a band-passed data string to evaluate the time dependent spectrum within a specific frequency band. For such a requirement, the spectrums within two transition zones $\lambda_{c1}^H < \lambda < \lambda_{c2}^H$ and $\lambda_{c1}^L < \lambda < \lambda_{c2}^L$ in the high and low frequency sides of the band, respectively, should not be seriously counted. In fact, as comparing with the band-passed data obtained by just dropping the spectrum associated to $\lambda < \lambda_{c1}^H$ and $\lambda_{c2}^L < \lambda$, these transition zones are employed to keep the generality of the band-passed data string. The direct method of dropping undesired modes from the Fourier spectrum has the possibility of missing the information reflecting the frequency variation of waveforms.

Monotonic Cubic Interpolation Method

For the sake of simplicity and to employ the data structure of a computer, it seems convenient to use the simple FFT algorithm whose data points are exactly equal to 2^m. Because a real data string may not have exactly the number of integer powers of 2 in uniform spacing form, a interpolation scheme should be employed to redistribute the points. In Ref.[4], a Hermite cubic interpolation between points (x_i, x_{i+1}) is employed.

$$\begin{aligned} &y(x) = c_3(x - x_i)^3 + c_2(x - x_i)^2 + c_1(x - x_i) + c_0 \\ &c_0 = y_i,\ c_1 = y_i^{'},\ s_{i+1/2} = (y_{i+1} - y_i)/(x_{i+1} - x_i) \\ &c_2 = \frac{3s_{i+1/2} - 2y_i^{'} - y_{i+1}^{'}}{x_{i+1} - x_i},\ c_3 = \frac{y_i^{'} + y_{i+1}^{'} - 2s_{i+1/2}}{(x_{i+1} - x_i)^2} \end{aligned} \tag{7}$$

In order to avoid spurious oscillation, Ref.[3] modified the M3A interpolation of Ref.[4] to give necessary limiter to slopes $y_i^{'}, y_{i+1}^{'}$ as

$$y_i' = \mathrm{sgn}(t_i)\min\left[\frac{1}{2}\left|p_{i-1/2}'(x_i) + p_{i+1/2}'(x_i)\right|, \max\left\{k\,|\,s_i\,|, \frac{k}{2}\,|\,t_i\,|\right\}\right]$$

$$\begin{aligned} &p_{i-1/2}'(x_i) = s_{i-1/2} + d_{i-1/2}(x_i - x_{i-1}), \\ &p_{i+1/2}'(x_i) = s_{i+1/2} + d_{i+1/2}(x_i - x_{i+1}), \\ &t_i = \min\mathrm{mod}\left[p_{i-1/2}'(x_i), p_{i+1/2}^{i}(x_i)\right], \\ &d_{i+1/2} = \min\mathrm{mod}\left[d_i, d_{i+1}\right], \\ &d_i = (s_{i+1/2} - s_{i-1/2})/(x_{i+1} - x_{i-1}),\ \ s_i = \min\mathrm{mod}\left[s_{i-1/.2}, s_{i+1/2}\right] \\ &k = 3, \quad \text{if } |\,s_{i+1/2}\,| >> |\,s_{i-1/2}\,|, \text{ or } |\,s_{i-1/2}\,| >> |\,s_{i+1/2}\,| \\ &\quad \geq 4, \quad \text{otherwise} \end{aligned} \tag{8}$$

At two ends, the Hyunk boundary condition will be employed [4]. As will be discussed later, this cubic interpolation might introduce too much artificial modification.

Fast Fourier Transform Algorithm

In terms of the iterative filter, the FFT algorithm of Ref.[3] is modified to be the following steps:

1. Employ the iterative filter to remove the non-sinusoidal and low frequency parts.
2. For the remaining high frequency part, choose zero crossing points at two ends. Use an interpolation method to find 0 points there.
3. Use the modified monotonic cubic interpolation of Ref.[3] to regenerate the data so that total number of points are of 2^m. For a smooth data string, more than one point should be located in the range between two successive data points of the original data string to reduce interpolation error. For an oscillatory data such as the turbulent data taken by an insufficient sampling rate, more than 4 points should be considered.
4. Perform an odd function mapping with respect to one end so that the final data point is doubled.
5. A simple and fast Fourier sine transform algorithm is employed to generate the desired spectrum.

Since the values are chosen at two ends, the penalty of shrinking the available data range can not be avoided. Note that the odd function mapping makes the perfect periodicity of the resulting data string.

The True Short Time Fourier Transforms

The authors believe that, if the above mentioned FFT algorithm is not employed and if the non-sinusoidal part is not removed, it is not easy to construct a true short time Fourier transform. The possible reason comes from the non-periodicity of the data string. This study will employ the following strategy to construct the short time Fourier transforms.

1. Identify all the zero crossing points, say x_i^0's, in terms of a searching procedure together with a suitable interpolation method.
2. After assign a proper window size ΔS, starting from the left end, define a series of successive windows T_i whose left end points are x_i^0. The right end point of window T_i is chosen at x_{i+k}^0, $k > 1$ and satisfies the criterion that $\left\|x_{i+k}^0 - x_i^0\right| - \Delta S\right|$ is minimum with respect to all positive integer k.
3. Perform the proposed FFT algorithm to evaluate the spectrum for each window T_i. Assume the resulting spectrum is valid at central point $x = (x_i^0 + x_{i+k}^0)/2$. The spectrum is interpreted by the frequency in terms of the following equation.

   $$f_n = n/[2(x_{i+k}^0 - x_i^0) \tag{9}$$

   where n is the mode number and the factor 2 in the denominator results from the odd function mapping procedure.

Note that, for a real data string, the window size $x_{i+k}^0 - x_i^0$ differs from each other and the resulting time frequency spectrum shows an oscillatory behavior obviously whenever the window size is not wide enough as will be shown below.

The Uncertainty Properties of Fourier Spectrum [7]

Define operators on time and frequency domains, respectively as $T = t$ and $F = -j(d/dt)$ on time domain where $j = \sqrt{-1}$. For a signal $Ae^{j\omega t}$, $F \cdot (Ae^{j\omega t}) = \omega Ae^{j\omega t}$, where $\omega = 2\pi f$. The commutation between these two operators is again an operator defined as

$$[T, F] = TF - FT = t(-j\frac{d}{dt}) - (-j\frac{d}{dt})t = j \tag{10}$$

Since the commutation is not zero, time and frequency do not commute. Thus, time and frequency cannot be measured independently and there is uncertainty between them which is

$$\Delta t \cdot \Delta\omega \geq 0.5\left|\langle [T, F] \rangle\right| = 0.5\left|\langle j \rangle\right| = 0.5 \tag{11}$$

In other words, if the short time Fourier transform is employed to resolve the local variation of spectrum with respect to time, the resolution is limited. In other words, a small window width (corresponding to a small Δt and good temporal resolution of local property), the accuracy of frequency is will be bad. On the other hand, a large widow width with worse temporal resolution leads to an accurate frequency resolution. It is well known that, for the Gabor transform, if a large window size is employed, the resulting Gabor coefficient plot is called the narrowband spectrogram which is used for good frequency resolution. On the other hand, the wideband spectrogram uses a small window size which allows good temporal resolution of signals. We will employ this principle to examine the property of the present short time Fourier transforms.

3. RESULTS AND DISCUSSIONS

In Ref.[3], there is not example with known function and exact spectrum to demonstrate the merit of the proposed FFT algorithm. For the sake of completeness, the following function is employed to explicitly examine the resulting error of the proposed FFT algorithm.

$$\begin{aligned} y(x) &= \bar{y}(x) + y_1(x) \\ \bar{y}(x) &= 1 + 2\bar{x} + \bar{x}^2/2 + 0.3\exp[-0.05x^2]\sin(6\pi\bar{x}) \\ y_1(x) &= 0.3\exp[-0.5\bar{x}^2](1+2\bar{x}+\bar{x}^2)\sin(32\pi\bar{x}) \\ &\quad + 0.2\sin(56\pi\bar{x}) + 0.4\sin(100\pi\bar{x}) \end{aligned} \tag{12}$$

where $\bar{x} = x/10$, $y(x)$ is the composite waveform (shown as the thin solid line in Fig.1), $\bar{y}(x)$ involves a non-sinusoidal and low frequency part (dotted line) and $y_1(x)$ is composed of two simple short waves and a complicated short wave (shown as thin solid line around $x = 0$). At first, the data is expressed in a uniform spacing in the range of $0 \le x \le 10$ so that the total number of point is 8192+1. The first 8192 points are employed to find the spectrum via a simple FFT program. After apply the iterative filter, the estimated smooth part is shown as long dashed line, and the high frequency part is the dashed line. Obviously, the estimated smooth part deviates from the given smooth part and their difference becomes significant around two ends. Figure.2 shows three spectrums: the thin solid line with diamond symbol is the spectrum of $y(x)$, the heavy solid line with delta symbol is the spectrum of $y_1(x)$ plus an odd function mapping with respect to the point at $x = 10$ so that the total width is doubled. The line with gradient symbol is the spectrum of the estimated high frequency part via the iterative filter. The low frequency error of $y(x)$'s spectrum is induced by both the non-sinusoidal part of $\bar{y}(x)$ and the non-periodic condition at two ends. If $\bar{y}(x)$ is removed from $y(x)$, the resulting spectrum does not involve the non-sinusoidal part and the error induced by $y(10) \ne y(0)$. The exact spectrum is achieved because $y(0) = y(10)$ and all the available conditions of derivatives at two ends, say $y'(0) = y'(10)$, $y''(0) = y''(10)$,… etc. are satisfied. It is seen that the estimated spectrum captures all the dominate modes. The small disagreement comes from the deviation of estimated smooth part from the original smooth part around two ends. Nevertheless, this test case shows that the present FFT algorithm does significantly reduce the spectrum error as compare with the result of classical FFT algorithm.

The authors had performed many test studies the results showed that, if the composite wave forms a beat, the resulting Fourier spectrum converges slowly for an improper choice of end points. In order to demonstrate such property, the following composite wave is examined as a first test case.

$$y(x) = \sin\frac{46\pi x}{24} + 0.9\sin\frac{50\pi x}{24} + 0.1\sin(2\pi x) \tag{13}$$
$$0 \le x \le 365 \text{ days}$$

The detailed plot the original data is shown in Fig.3 which clearly in form of beat wave. Figures. 4 show two pictures of local short time Fourier spectrum whose approximate segment intervals are 30 and 225 days, respectively. From these two figures, it is clear that the local short time Fourier series evaluated by end points located at successive zero crossing points involves certain degree of error. For convenient, the line connecting local maximum points are considered as the upper limiting envelope and that formed by the local minimum points is designed as the lower limiting envelope. The approximate radius is thus defined by the vertical distance between lower and upper limiting envelopes. A careful inspection upon Figs.3 and 4 reveals that the data around $r \approx$ minimum is not of regular shape and the spectrums corresponding to end points around these regions shown a local splitting property. For a short expansion period of 30 days, the error is rather serious and even shows a multiple solution. According to the uncertainty principle of Eq.(11), a long expansion period will give a small spectrum error. However, the result of Fig.4b which uses the expansion range of 225 days, the multiple solution still present. In other words, the zero crossing point around minimum radius will introduce a maximum spectrum error. Suppose the end points are only located around $r \approx$ maximum, the resulting spectrums with periods of 32 and 96 days are shown in Figs.5a and 5b, respectively. This test case clearly shows that, one has better to choose end points at zero crossing points where the composite wave has a regular and smooth shape. Any small wavy shape around a zero crossing point will lead to spectrum error.

The second test case is the tide wave data of Hobihu habor at Pin-Dong (in south part of Taiwan). The data range runs from January 1/2001 to December 31/2001. Figure.6a shows the original data (around sea level = 1000 mm) and the whole day tide data (around sea level = 0 mm). The latter wave is obtained by employing the iterative filter with $m = 30$ iterations and $\sigma = 4$ days to remove the low frequency part and $m = 200$ iterations and $\sigma = 0.35$ days to exclude the high frequency part. The resulting whole day tide wave data shows a beat wave of period 15 days which is about half of a lunar month as shown in the detailed plot of Fig.6b. This is a practical data with a complicated composite beat wave and is a band-passed data filtered from the original data. The resulting spectrum is shown in Fig.7.

In Fig.6, the data around regions of minimum radius is not a regular variation and is even more complicated than that shown in Fig3. According to the conclusion of the previous test case, it is better to exclude those spectrum estimated by the segments whose end points located around this range. With the restriction of this rule, a series of two-dimensional amplitude plots of wave width vs. time are shown in Fig.8 with different window size. Because of many waves combining together, the window sizes (determined by zero-crossing points) are not uniform. For the sake of easy plotting the result, the data in vertical direction is redistributed via linear interpolation to smooth the result. These spectrums show that the increase of expansion range does decrease the spectrum error and the limit is the spectrum shown in Fig.7. A first glance upon Fig.6b shows that the upper and lower limiting envelopes of the beat wave is not smooth. Therefore, the resulting dominate modes are not in form of a perfectly shaped impulses. In other words, resulting bands of dominate modes show a faded character.

The third test case is the voice data of “hello” of an 10 years old child. The data and spectrum are shown in Fig.9a and 9b, respectively. In this case the band passed filter is obtained by two stages: $\sigma = 1$ second, $m = 1$ iteration to remove low frequency part and $\sigma = 0.001$ seconds, $m = 200$ iterations to remove the high frequency part. In Fig.10a-d, the approximate window widths are 0.016, 0.02, 0.031, and 0.061 seconds, respectively. For a speech signal, Fig.10 can be considered as a series of spectrograms with different window sizes that are corresponding to different window widths. A careful inspection upon all the figures of Fig.10 reveals that the spectrogram with window width 0.02 second (Fig.9b) gives a best resolution. The spectrogram with window width 0.016 second (Fig.10a) gives an excellent resolution of rapid frequency changing part but the band width of the constant frequency part is bigger than that of Fig.10b. On the other hand, those use window widths 0.031 and 0.062 seconds obtain a good resolution at frequency constant part but show a serious dilution of band resolution in rapid frequency changing part. In other words, this test case shows that the present short time Fourier transforms follows the uncertainty principle of Ref.[7].

4. CONCLUSIONS

The simple strategy of FFT algorithm with small spectrum error and the iterative filter are employed to generate a true short time Fourier transforms. A test case of three known sine waves with fixed frequency is employed to examine the effect of put two ends points at different location of a beat wave. The result shows that one should not choose zero crossing points around which approximate radius of the beat attains minimum value. A tide wave data is employed the capability of resolving temporal behavior of a composite wave with a short widow width. The other test case shows that the present transform is restricted by the uncertainty principle of most time-frequency analysis tools. It is believed that this short time Fourier transforms is an convenient tool to examine a time series data with complex structure.

ACKNOWLEDGEMENT

This work is supported by the National Science Council of Taiwan under the grant number NSC-93 -2212-E006-037.

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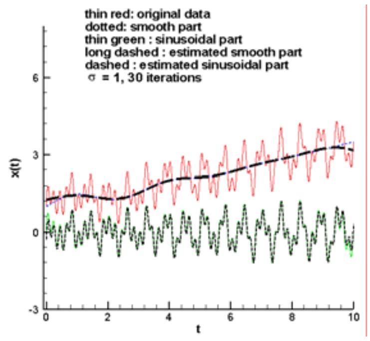


Fig.1 The test function: original composite wave is thin solid line, given smooth part is the dotted line; given high frequency part is thin solid line around $x(t)=0$; the long dashed line is the smooth part estimated by the iterative filter with $\sigma=1, m=30$; and the dashed line around $x(t)=0$ is the estimated high frequency part.

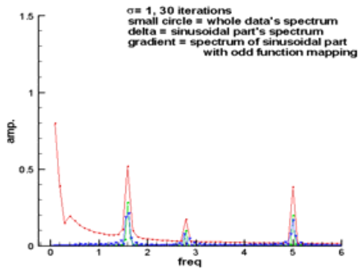


Fig.2 The resulting spectrums: diamond symbol is the spectrum evaluated from $y(x)$;delta symbol is the exact spectrum; and the gradient symbol is the spectrum estimated by the high frequency part of Fig.1.

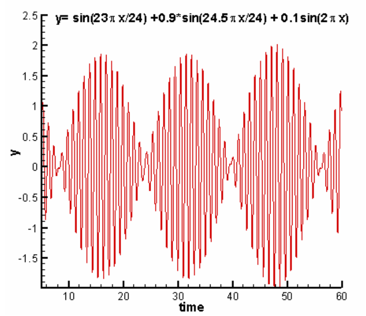
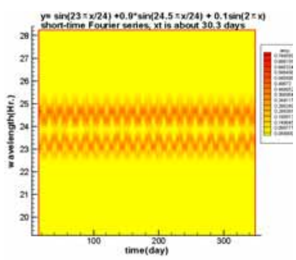


Fig.3 The detailed plot of the test function of Eq.(13).

(4a)



(4b)

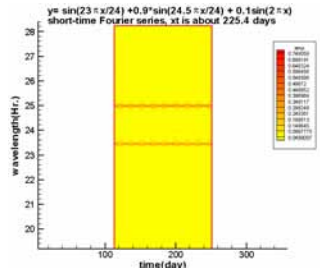
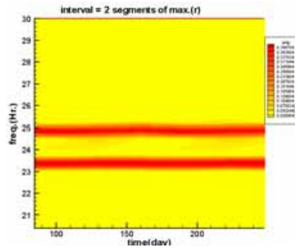


Fig.4 The resulting spectrums: (a) $\mathrm{T_{total}} \approx 30$ days and (b) $\mathrm{T_{total}} \approx 225$ days.

(5a)



(5b)

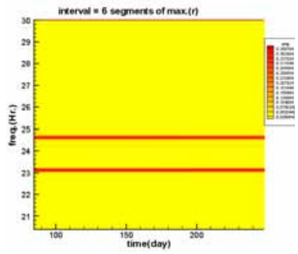
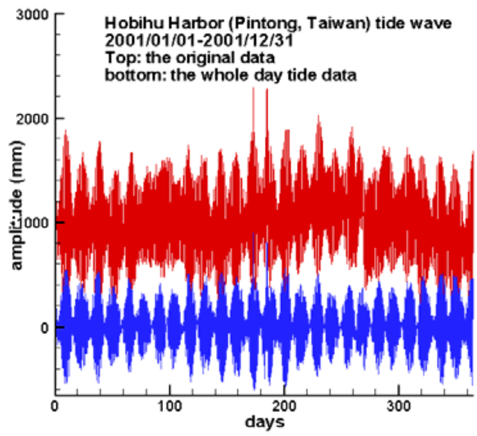


Fig.5 The resulting spectrums: (a) $\mathrm{T_{total}} \approx 32$ days and (b) $\mathrm{T_{total}} \approx 96$ days with two end points located at $r \approx r_{\max}$.

(6a)



(6b)

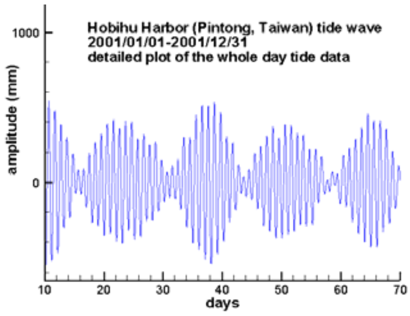


Fig.6 The tide wave data of Hobihu habor (Pingtung, Taiwan): (a) the upper line around 1000 mm is the original data; the line around 0mm is the whole day tide data (filtered by $\sigma = 4$, 30 iterations and $\sigma = 0.35$, 200 iterations) and (b) the detailed plot of the whole day tide data.

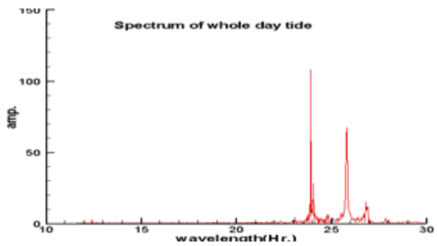
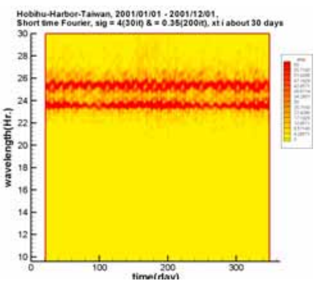
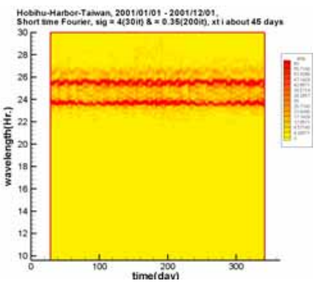


Fig. 7 The spectrum of the whole day tide data of Fig.6 of Fig.6.

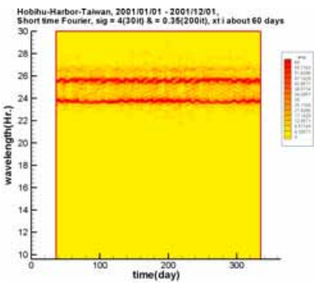
(8a)



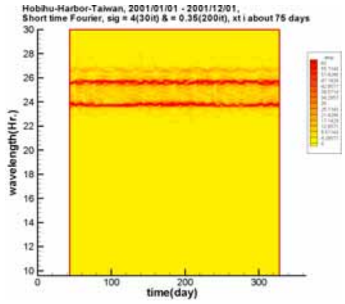
(8b)



(8c)



(8d)



(8e)

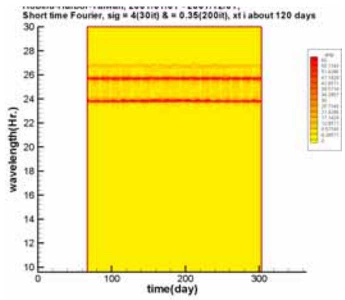
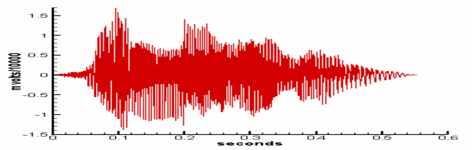


Fig.8The two-dimensional amplitude plots of short time Fourier transform with different interval sizes, evaluated by the band-passed data of Fig.6: (a) width $\approx$ 30 days; (b) 45 days; (c) 60 days; (d) 75 days; (e) 90 days; and (f) 120 days.

(9a)



(9b)

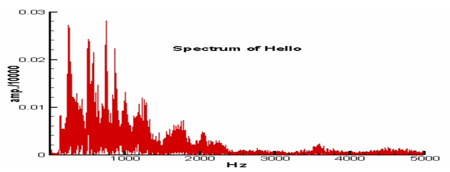
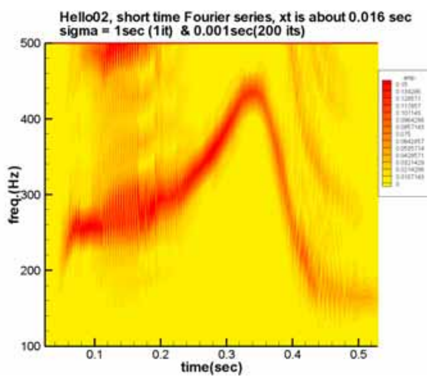
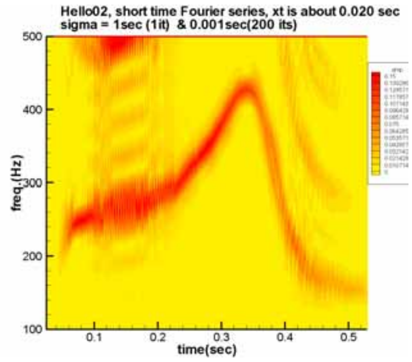


Fig.9 The data and spectrum of three voice “hello”: (a) the raw data of voice “hello”; and (b) the spectrum.

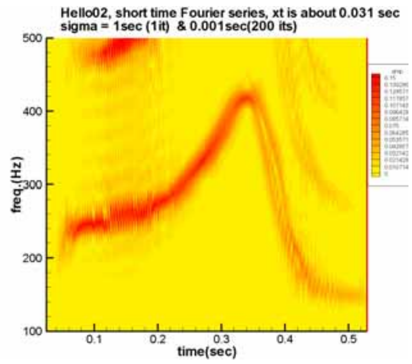
(10a)



(10b)



(10c)



(10d)

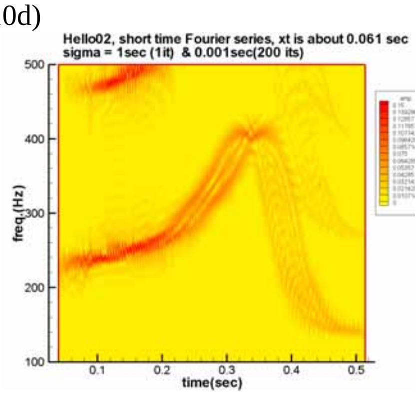


Fig.10 The two-dimensional amplitude plots of low frequency part of the voice “hello”: (a) interval length is about 0.016 seconds; (b) 0.02 seconds; (c) 0.031 seconds; and (d) 0.061 seconds. The band passed data is generated by steps: $\sigma = 1$, $m = 1$ to remove low frequency part and $\sigma = 0.001$ seconds, $m = 200$ iterations to remove the high frequency part.

時變數據的二維短時間間格傳數轉換法之研究

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摘要

本文使用低誤差的簡易快速傅立葉轉換式配合疊代型濾波器，發展出真正的短時間間格之傅立葉轉換式以求出二維的時頻圖。將一串時變數據產生時頻圖之前，先用疊代型濾波器移除非週期和低頻部份。經取得穿越 0 點位置後配合內插法求得高頻部份之 0 點。在設定好每一數據段的窗口寬後，從最左邊之 0 點起，找出其它 0 點使其距離最接近於設定窗口寬度，將這左右兩點當成最左邊的數據段。準此原則，往右找，可依次找到所要的各個數據段，最右邊的數據串可令其稍長。針對每一數據段使用簡易快速傅立葉轉換式找其頻譜，並將之當成每一數據串中點的頻譜，即可得到二維時頻圖。本文應用三個已知頻率的正弦波之組合說明必需跳過零點在組合波的近似振幅達到最小值附近的解，以減小誤差。屏東後壁湖漁港之潮汐水位資料，發現短窗口寬之時頻圖之頻率解析度差，看可以補捉到時變資料，加大窗口寬度雖增加頻率解析度，但時變解析度較差。本文也以一個英文字”hello”的聲音訊號，顯示本文時頻圖在時間和頻率方向的解析能力。

Keywords: 時頻圖，快速傅立葉轉換式。